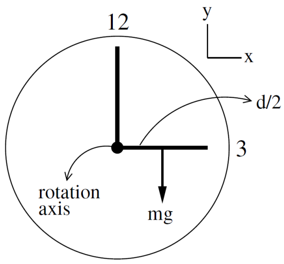
P1 [2009-2010 Spring - Final]: The hour hand and the minute hand of Big Ben, a famous tower clock in London, are 2.7 m and 4.5 m long and have masses of 60 kg and 100 kg, respectively. Treating the hands as long *uniform* thin rods, determine the total torque due to the weight of Big Ben's hands about the axis of rotation when the time reads (a) 03:00, (b) 06:00, (c) 09:00, and (d) 03:33 PM, as in the figure. [The last part (d) is bonus.]

SOLUTION: $d = 2.7\ \text{m}$, $D = 4.5\ \text{m}$, $m = 60\ \text{kg}$, $M = 100\ \text{kg}$

(a) For 03:00 o'clock, check first the figure on the right.



$$\vec{\tau}_{\text{minute}} = 0,$$

$$\vec{\tau}_{\text{hand}} = Fr_{\perp} = mg \cdot \frac{d}{2}\left(-\hat{\text{k}}\right) = -(60)(9.8)\left(\frac{2.7}{2}\right)\hat{\text{k}}$$

$$\Rightarrow \quad \vec{\tau}_{\text{hand}} \approx \left(-794\,\hat{\text{k}}\right)\text{N}\cdot\text{m}$$

$$\therefore \quad \vec{\tau}_{\text{total}} = \vec{\tau}_{\text{minute}} + \vec{\tau}_{\text{hand}} \approx \left(-794\,\hat{\text{k}}\right)\text{N}\cdot\text{m} \qquad ☺$$

(b) At 06:00 o'clock, both $\vec{\tau}_{\text{minute}}$ and $\vec{\tau}_{\text{hand}}$ are zero, so that

$$\vec{\tau}_{\text{total}} = 0 \qquad ☺$$

(c) The 09:00-o'clock situation is similar to the 09:00-o'clock situation, there will be only a sign change:

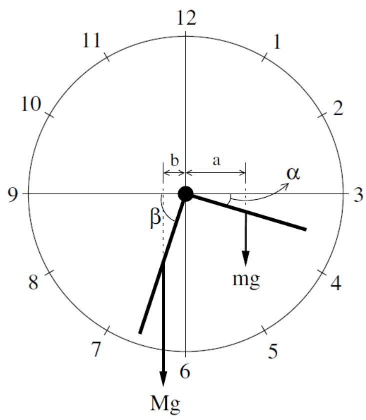
$$\vec{\tau}_{\text{total}} \approx \left(+794\,\hat{\text{k}}\right)\text{N}\cdot\text{m} \qquad ☺$$

(d) [BONUS] Check the below figure. We have

$$\alpha = \frac{(33\ \text{min})(90^{\text{o}})}{(3\times 60\ \text{min})} = 16.5^{\text{o}}$$

and

$$\beta = \frac{(12\ \text{min})(90^{\text{o}})}{15\ \text{min}} = 72^{\text{o}}$$



$$\vec{\tau}_{\text{minute}} = Mg \cdot b = Mg\frac{D}{2}\cos\beta\left(+\hat{\text{k}}\right)$$

$$\Rightarrow \quad \vec{\tau}_{\text{minute}} = (100)(9.8)\left(\frac{4.5}{2}\right)\cos 72^{\circ}\ \hat{\mathrm{k}}$$

$$\Rightarrow \quad \vec{\tau}_{\text{minute}} = \left(681.38\,\hat{\mathrm{k}}\right)\ \mathrm{N \cdot m}$$

$$\vec{\tau}_{\text{hand}} = mg \cdot a = mg\frac{d}{2}\cos\alpha\left(-\hat{\mathrm{k}}\right)$$

$$\Rightarrow \quad \vec{\tau}_{\text{hand}} = -(60)(9.8)\left(\frac{2.7}{2}\right)\cos 16.5^{\circ}\ \hat{\mathrm{k}} = \left(-761.11\,\hat{\mathrm{k}}\right)\ \mathrm{N \cdot m}$$

$$\therefore \quad \vec{\tau}_{\text{total}} = \vec{\tau}_{\text{minute}} + \vec{\tau}_{\text{hand}} = \left(681.38\,\hat{\mathrm{k}}\right) + \left(-761.11\,\hat{\mathrm{k}}\right) \approx \left(-79.7\,\hat{\mathrm{k}}\right)\ \mathrm{N \cdot m} \qquad ☺$$

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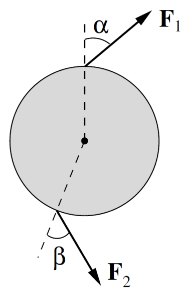
P2 [2009-2010 Summer - Final]: In the below figure is a uniform disk that can rotate around its center. The disk has a radius of $r = 4\,\text{cm}$ and a mass of $m = 0.4\,\text{kg}$ and is initially at rest. Starting at $t = 0$, two forces are applied to the rim as indicated, so that at time $t = 2\ \text{s}$ the disk has an angular velocity of ω = 340 rad/s counterclockwise. If $\alpha = \beta = 50^{\circ}$, and force $\vec{F}_1$ has a magnitude of $4\,\text{N}$, what is the magnitude of force $\vec{F}_2$? (The rotational inertia of the disk around its center is $\frac{1}{2}mr^2$.)



SOLUTION $r = 4\,\text{cm} = 0.04\ \text{m}$, $m = 0.4\,\text{kg}$, $F_1 = 4\,\text{N}$, $t = 2\ \text{s}$,

$\omega_0 = 0$, $\omega = 340\ \text{rad/s}$, $\alpha = \beta = 50^{\circ}$, $I = \frac{1}{2}mr^2$

$$\omega = \omega_0 + \alpha t = 0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{\omega}{t}$$

$$\tau = I\alpha \quad \Rightarrow \quad -rF_1\sin\alpha + rF_2\sin\beta = \tfrac{1}{2}mr^2 \cdot \frac{\omega}{t}$$

Since $\alpha = \beta$,

$$r\sin\alpha\left(F_2 - F_1\right) = \frac{m\omega r^2}{2t}$$

whence

$$\Rightarrow \quad F_2 = \frac{mr\omega}{2t\sin\alpha} + F_1 = \frac{(0.4)(0.04)(340)}{(2)(2)\sin 50^{\circ}} + 4 \quad \Rightarrow \quad F_2 \approx 5.78\ \mathrm{N} \qquad ☺$$

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P3 [2010-2011 Summer - MT3][E]: The dimension of **torque** is the same as that of (a) impulse, (b) energy, (c) momentum, (d) none of these. *Verify your answer.*

SOLUTION: Torque is defined as $\vec{\tau} = \vec{r} \times \vec{F}$ and has dimension

$$[\tau] = \text{m} \cdot \text{N} = \text{m} \cdot \text{kg} \cdot \text{m/s}^2 = \text{kg} \cdot \text{m}^2/\text{s}^2$$

(a) Impulse is defined as $\vec{J} = \vec{F}\,\Delta t$ and has dimension

$$[J] = \text{N} \cdot \text{s} = \text{kg} \cdot \text{m/s}^2 \cdot \text{s} = \text{kg} \cdot \text{m/s}$$

(b) Kinetic energy, for example, is defined as $K = \frac{1}{2}mv^2$ and has dimension

$$[K] = \text{kg} \cdot (\text{m/s})^2 = \text{kg} \cdot \text{m}^2/\text{s}^2$$

(c) Momentum is defined as $\vec{p} = m\vec{v}$ and has dimension

$$[p] = \text{kg} \cdot \text{m/s}$$

The correct answer is (b). ☺

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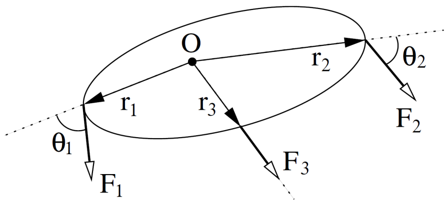
P4 [2010-2011 Summer - Final][E]: The plate in the figure is pivoted at O, and three forces act on it as shown. If $r_1 = 1.3$ m, $r_2 = 2.15$ m, $r_3 = 1.1$ m, $F_1 = 4.2$ N, $F_2 = 4.9$ N, $F_3 = 3.9$ N, $\theta_1 = 75^\circ$, and $\theta_2 = 60^\circ$, what is the net torque about the pivot?



SOLUTION: Note that the extension of $\vec{F}_3$ goes through the pivot point so that it does not contribute to the net torque. Then,

$$\tau_{\text{net}} = r_1 F_1 \sin\theta_1 - r_2 F_2 \sin\theta_2 = (1.3)(4.2)\sin 75^\circ - (2.15)(4.9)\sin 60^\circ \approx -3.85 \text{ N} \cdot \text{m}$$

where the negative sign shows that the net torque is in the clockwise direction. ☺

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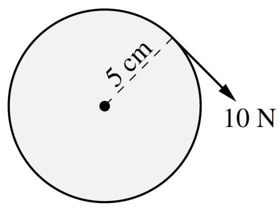
P5 [2011-2012 Spring - Final][E]: A 10 N-force is applied *tangentially* to a disk that is free to rotate about an axis through its center. Calculate the magnitude of the torque about the rotation axis.



SOLUTION: $F = 10$ N, $r = 5 \text{ cm} = 0.05$ m

$\tau = rF \sin\theta = rF \sin 90^{\circ} = rF = (0.05)(10) = 0.50 \text{ N}\cdot\text{m}$ ☺

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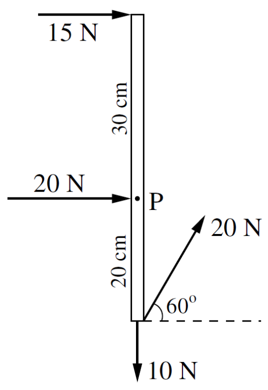
P6 [2014-2015 Fall - Final][E]: Four forces are applied to a rod that can rotate about a perpendicular rotation axis through point P, as shown in the figure. Find the magnitude and direction, *clockwise* or *counterclockwise*, of the resultant torque about the rotation axis.



SOLUTION: $F_1 = 15 \text{ N}$, $F_2 = 20 \text{ N}$, $F_3 = 10 \text{ N}$, $F_4 = 20 \text{ N}$, $\theta = 60^{\circ}$, $d_1 = 30 \text{ cm} = 0.3 \text{ m}$, $d_4 = 20 \text{ cm} = 0.2 \text{ m}$

$\vec{F}_2$ and $\vec{F}_3$ do not produce any torque because their directions pass through the rotation axis.

$$\tau_{\text{net}} = -F_1 d_1 + F_4 \cos\theta \cdot d_4 = -(15)(0.3) + (20)(\cos 60^{\circ})(0.2) = -2.50 \text{ N}\cdot\text{m}$$ ☺

where the minus sign indicates that the net torque is in the clockwise direction. ☺

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