CHAPTER 1 | MEASUREMENT

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Outline

Introduction

- **Experiments**: performed for the **measurements** & **comparisons** of **(physical) quantities** (length, mass, time, temperature, current, force, pressure, velocity, acceleration, etc.)

- Physical quantities can be expressed in terms of a few **base quantities** (length, mass, time, current, temperature, etc).

  Example: Force $\rightarrow$ "mass times length per time-squared"

- **Unit**: a special name for the measure of a quantity.

  Examples: meter (m), second (s), meter per second-squared (m/s^2).

SI Units

- **The SI unit system** or **metric system**:

Quantity	Unit Symbol	Unit Name
length	m	meter
mass	kg	kilogram
time	s	second
electric current	A	ampere
temperature	K	kelvin
luminous intensity	cd	candela
amount of substance	mol	mole
2D plane angle	rad	radian
3D solid angle	sr	steradian

- **Base quantities** $\rightarrow$ **base units**
- **Derived units**: defined in terms of base units.

  Example: the SI unit of force $\rightarrow$ the **newton** (N)

  $$1 \text{ newton} = 1 \text{ N} = 1 \text{ kg} \cdot \text{m}/\text{s}^2$$

- **Warning!**: throughout this course, we will work with SI units; learn them by heart!

Atomic Mass Unit (amu)

- **Atomic mass unit (amu)**: used to express atomic and molecular masses; defined as *one twelfth of the mass of the carbon*-12 *atom*.

  The conversion between amu and kg:

$$1 \text{ amu} = 1.660\,538\,86 \times 10^{-27} \text{ kg}.$$

Scientific Notation $\rightarrow$ powers of 10

$$
\begin{aligned}
&\vdots \\
10^{-5} &= 0.00001 \\
10^{-4} &= 0.0001 \\
10^{-3} &= 0.001 \\
10^{-2} &= 0.01 \\
10^{-1} &= 0.1 \\
10^{0} &= 1 \\
10^{1} &= 10 \\
10^{2} &= 100 \\
10^{3} &= 1000 \\
10^{4} &= 10,000 \\
10^{5} &= 100,000 \\
&\vdots
\end{aligned}
$$

- Numbers in the form

$$a \times 10^n$$

are said to be written in **scientific notation**, where a is between 1 and 10.

Examples:

$$\begin{aligned} 10\,419\,500\,000 &= 1.04195 \times 10^{10} \\ 0.000262 &= 2.62 \times 10^{-4} \end{aligned}$$

- In the general form 10^n, n is called the **exponent** of 10.
- **Decimal numbers** (or **floating numbers**) $\rightarrow$ 123.45678
- **Digit**, **decimal point**, and **decimal places**.

- Scientific notation in your calculators:

  6.567E9 and 0.273E-4

  "E" $\rightarrow$ **exponent of 10.**

  Translate these as

$$\begin{aligned} 6.567\text{E}9 &= 6.567 \times 10^{9} \\ 0.273\text{E-}4 &= 0.273 \times 10^{-4} \end{aligned}$$

- Multiplication and division in scientific notation:

$$10^m \times 10^n = 10^{m+n}$$

$$\frac{10^m}{10^n} = 10^{m-n}$$

Examples

- Decimal numbers and their forms in scientific notation:

$$
\begin{aligned}
-0.000298564 &= -2.98564 \times 10^{-4} \\
0.0739458 &= 7.39458 \times 10^{-2} \\
82\,789 &= 8.2789 \times 10^{4} \\
-1\,928.254 &= -1.928254 \times 10^{3} \\
225\,568\,252 &= 2.25568252 \times 10^{8}
\end{aligned}
$$

Examples

- Multiplication and division in scientific notation:

$$\left(4 \times 10^2\right)\left(3 \times 10^4\right) = 12 \times 10^6 = 1.2 \times 10^7$$

$$\left(3.1 \times 10^{-2}\right)\left(6.2 \times 10^7\right) = 19.22 \times 10^5 = 1.922 \times 10^6$$

$$\frac{6.3 \times 10^{-3}}{3 \times 10^{-7}} = 210 \times 10^2 = 2.1 \times 10^4$$

$$\frac{2.2 \times 10^{-5}}{\left(8.8 \times 10^{-3}\right)\left(25 \times 10^{-1}\right)} = 0.01 \times 10^{-1} = 1 \times 10^{-3}$$

- The **prefixes** that shall be employed frequently throughout the course; just memorize them!

Power	Prefix	Symbol	Power	Prefix	Symbol
10^{-15}	femto	f	10^{-2}	centi	c
10^{-12}	pico	p	10^{3}	kilo	k
10^{-9}	nano	n	10^{6}	mega	M
10^{-6}	micro	μ	10^{9}	giga	G
10^{-3}	milli	m	10^{12}	tera	T

- Each prefix in this table represents a certain power of 10:

$$
\begin{aligned}
2.34 \text{ km} &\equiv 2.34 \text{ kilometers} = 2.34 \times 10^{3} \text{ m} \\
6.41 \text{ ms} &\equiv 6.41 \text{ milliseconds} = 6.41 \times 10^{-3} \text{ s}
\end{aligned}
$$

- **Warning!**: in these notes and in our textbook, all mathematical symbols are written in *italic* letters and all numbers and units in plane letters:

  Example: $F = 4.56$ N

- Thus, the meaning of 2.23 mg is not to be confused with that of 2.23 mg.

Changing Units

- **Chain-link conversion**: a method to convert a given unit into any other unit.

  Example: since 1 day $=$ 24 hours, we can write

$$\frac{1 \text{ day}}{24 \text{ hours}} = 1 \quad \text{or} \quad \frac{24 \text{ hours}}{1 \text{ day}} = 1$$

  These are **conversion factor**; just insert them wherever you like in an equation.

Example

How many seconds are in one year?

SOLUTION:

$$1 \text{ year} = 1 \,\cancel{\text{year}} \times \frac{365 \,\cancel{\text{days}}}{1 \,\cancel{\text{year}}} \times \frac{24 \,\cancel{\text{h}}}{1 \,\cancel{\text{day}}} \times \frac{60 \,\cancel{\text{min}}}{1 \,\cancel{\text{h}}} \times \frac{60 \text{ s}}{1 \,\cancel{\text{min}}} = 3.1536 \times 10^7 \text{ s}$$

Example

Perform the following conversion

$$a = 90.36\,\frac{\mathrm{km/h}}{\mathrm{s}} = ?\ \mathrm{m/s^2}.$$

SOLUTION:

$$\begin{aligned} a &= 90.36\,\frac{\mathrm{km/h}}{\mathrm{s}} \\ &= 90.36\,\frac{\cancel{\mathrm{km}}}{\cancel{\mathrm{h}}\cdot\mathrm{s}} \times \frac{1\times10^3\ \mathrm{m}}{1\ \cancel{\mathrm{km}}} \times \frac{1\ \cancel{\mathrm{h}}}{60\ \cancel{\mathrm{min}}} \times \frac{1\ \cancel{\mathrm{min}}}{60\ \mathrm{s}} \\ &= 25.1\ \mathrm{m/s^2} \end{aligned}$$

Example

The density of aluminum, near room temperature, is $2.70\ \mathrm{g \cdot cm^{-3}}$. Express this density in units of $\mathrm{kg \cdot m^{-3}}$.

SOLUTION:

$$\begin{aligned} 2.70\ \mathrm{g \cdot cm^{-3}} &= 2.70\ \frac{\cancel{\mathrm{g}}}{\cancel{\mathrm{cm}^3}} \times \frac{1\ \mathrm{kg}}{1 \times 10^3\ \cancel{\mathrm{g}}} \times \left(\frac{1\ \cancel{\mathrm{cm}}}{1 \times 10^{-2}\ \mathrm{m}}\right)^3 \\ &= 2700\ \mathrm{kg/m^3} = 2700\ \mathrm{kg \cdot m^{-3}} \end{aligned}$$

- Units in conversions must obey the ordinary algebraic rules.
- Note that cm^3 means $(cm)^3$.

Rounding

- An example:

$$
\begin{aligned}
8.5014936 &= 8.501493|\mathbf{6} \approx 8.501494 \\
8.5014936 &= 8.50149|\mathbf{3}6 \approx 8.50149 \\
8.5014936 &= 8.5014|\mathbf{9}36 \approx 8.5015 \\
8.5014936 &= 8.501|\mathbf{4}936 \approx 8.501 \\
8.5014936 &= 8.50|\mathbf{1}4936 \approx 8.50 \\
8.5014936 &= 8.5|\mathbf{0}14936 \approx 8.5 \\
8.5014936 &= 8.|\mathbf{5}014936 \approx 9
\end{aligned}
$$

Significant Figures

- All measured values on earth are subjected to unawoidable **(experimental) uncertainties**!!!

  Example: Suppose that you are to measure the dimensions of a rectangular prism and that the meter stick you use has an accuracy of ± 1 mm $= \pm 0.1$ cm. When you measure the length of the prism as 21.3 cm, the actual value is between 21.2 cm and 21.4 cm.

- Suppose that you measured the length, width, and height of the rectangular prism as

$$\ell = 21.3\,\text{cm} \pm 0.1\,\text{cm},\; w = 18.5\,\text{cm} \pm 0.1\,\text{cm},\; \text{and } h = 9.7\,\text{cm} \pm 0.1\,\text{cm}$$

where the former two have *three* and the latter has *two* significant figures.

- The thumbrule:

the first estimated digit, which imparts the experimental uncertainty, is contained in significant figures.

- The volume of this prism:

$$V = \ell wh = (21.3 \text{ cm})(18.5 \text{ cm})(9.7 \text{ cm}) = 3822.285 \text{ cm}^3$$

This result is nonsense! For, with its seeming seven significant figures, this result is more accurate than the given data.

- Multiplication and/or division:

> The number of significant figures in the final result must be the same as the *least* number of significant figures in the given data (which corresponds to the *least accurate* of the given quantities).

- Then, we round the result for the volume of the above prism to *two* significant figures:

$$V = (21.5\ \text{cm})(18.5\ \text{cm})(9.7\ \text{cm}) = 3822.285\ \text{cm}^3 \approx 3800\ \text{cm}^3$$

Actual volume is between

$$(21.2\ \text{cm})(18.4\ \text{cm})(9.6\ \text{cm}) \approx 3700\ \text{cm}^3$$

and

$$(21.4\ \text{cm})(18.6\ \text{cm})(9.8\ \text{cm}) \approx 3900\ \text{cm}^3.$$

- Consider a measured distance

$$d = 4500 \text{ m}$$

How many significant figures does this number have? Two? Three? Four?

- What about another quantity:

$$t = 0.0034 \text{ s}$$

It can be 0.00340 s or 0.003400 s; which is correct?

- To avoid such difficulties, use scientific notation.

  For the above examples, a two-significant figure distance value as $d = 4.5 \times 10^2$ m and a four-significant figure time value as $t = 3.400 \times 10^{-2}$ s.

- Now the rule:

> Except front-zeros that locate the decimal point, we count any reliably known digit as a **significant figure.**

- Adding and subtracting quantities:

> The number of decimal places of the final result and the *smallest* number of decimal places of any term in the process should match.

- *Examples*:

$$
\begin{aligned}
35 + 2.3 &= 37 \quad (\textit{not}\ \ 37.3) \\
2 + 0.25 &= 2 \quad (\textit{not}\ \ 2.25) \\
0.0008 + 2.7002 &= 2.7010 \quad (\textit{not}\ \ 2.701)
\end{aligned}
$$

Convention - I

Throughout these lecture notes, I will always assume that all the given quantities are accurate enough to result in answers with *three* significant figures.

Example: you will understand a distance of 10 m as 10.0 m or a force of 2 N as 2.00 N.

Convention - II

I will always keep at least *four* decimal places in the form of 1.2345 in all mid-results and will use them in the following procedures. Only at the very end will I give the main answer with *three* significant figures.

Convention - III

I will always take for granted the well-known constants and conversion factors with *three* significant figures.

Examples

$1 \text{ in.} = 2.54 \text{ cm}$	(in.: inch)
$1 \text{ ft} = 30.5 \text{ cm}$	(ft: foot)
$1 \text{ mi} = 1610 \text{ m}$	(mi: mile)
$c = 3.00 \times 10^{8} \text{ m/s}$	(speed of light)
$g = 9.80 \text{ kg} \cdot \text{m/s}^2$	(gravitational acceleration)
$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$	(Avogadro number)
$e = 1.60 \times 10^{-19} \text{ C}$	(elementary charge)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(proton mass)
$\varepsilon_0 = 8.85 \times 10^{-12} \text{ N} \cdot \text{m}^2/\text{C}^2$	(permittivity constant)

Order-of-Magnitude Calculations

- In scientific notation, the power of ten is called its **order of magnitude**.

  Example: the order of magnitude of

  $m_1 = 6.2 \times 10^2$ kg is 2

  and that of

  $m_2 = 3.7 \times 10^4$ kg is 4.

- ***Nearest* order-of-magnitude calculations**: are made for an estimated answer, for a preliminary result; are based on some assumptions and give an approximate result.

  Example: the nearest order of magnitude of

  $m_1 = 6.2 \times 10^2$ kg is 3

  and that of

  $m_2 = 3.7 \times 10^4$ kg is 4.

- An order-of-magnitude calculation is reliable within a factor of 10.

Example

We try to approximately determine the thickness of one single sheet of a 1070-page book with the help of a ruler.

I measure the thickness of this 535-sheet book as about 3.2 cm. Then, to the nearest order of magnitude, the thickness of one single sheet is

$$\text{thickness} = \frac{3.3 \times 10^{-2}\ \text{m}}{535\ \text{sheet}} \approx 6 \times 10^{-5}\ \text{m/sheet} = 10^{-4}\ \text{m/sheet}$$

Not bad at all!!!

Density

- **Density** ρ of a substance:

$$\rho = \frac{m}{V} \qquad \left(\mathrm{kg/m^3}\right)$$

- The "human scale" density of water:

$$\rho_{\text{water}} = 1.00\ \mathrm{g/cm^3}$$

Example: Platinum (mercury) is about 21 (14) times denser than water.

Thank you for your attention (and for your patience)!